Vectors – Teacher notes

Activity description

In this activity, basic manipulation of vectors in component form is reviewed and then vectors are used to solve two-dimensional problems based on real-life situations. These problems involve the use of the uniform acceleration formulae and Newton's Laws of Motion in vector form. Students then tackle a number of questions of a similar level of complexity to the worked examples.

Suitability

Level 3 (Advanced)

Time

1–2 hours

Resources

Student information sheet, slideshow

Equipment

Calculators

Key mathematical language

Scalar, vector, component, perpendicular, magnitude, direction, resultant, force, mass, acceleration

Notes on the activity

The first section of the Student information sheet is a summary of how to manipulate vectors given in component form, including finding magnitude and direction, adding and subtracting vectors, and multiplying a vector by a scalar.

This is followed by brief notes on the constant acceleration equations, momentum, and Newton's Laws of Motion. Some of these equations and laws are used in the worked examples to show how vectors can be applied in real-life contexts.

Students are then asked to work through a variety of problems.

The slides also give the content of the notes and worked examples.

These resources are for revision at the end of the course. They could also be used to introduce some of the concepts, but you may need to split the work into sections and provide extra examples and problems. Students need to be familiar with the use of perpendicular unit vectors **i** and **j**.

During the activity

The worked examples in the Information sheet and slideshow are highly detailed. Depending on the previous experience of the students, the examples could either be worked through as a whole class demonstration, or discussed by the students in pairs or small groups.

Alternatively, the questions in the slides could be a starting point for discussion in which ideas in the Information sheet could be drawn out.

Points for discussion

If students are struggling with the column vector notation, it might be worth drawing on the representation of a vector as a right-angled triangle using some of the discussion questions listed below.

- What is the advantage of using a right-angled triangle to represent a vector?

- How are the triangle representations of scalar multiples of a vector related?

- How does using perpendicular components of vectors help you to answer the questions?

- How have you used the fact that **i** and **j** are perpendicular unit vectors?

- In each question, the directions **i** and **j** were specified. Why was this necessary? Did the choice of directions of **i** and **j** affect the problem?

- What other scenarios can you think of which could be tackled using vectors in component form?

Extensions

There is some repetition in these questions. Students may notice this from their algebraic work, or through the diagrams they draw to help them solve the problems. The second extension activity asks students to look for similarities between the questions or the techniques or diagrams they've used.

If students have already studied the motion of a projectile, a possible extension activity is to reconcile the method used to solve Question 7 in this activity with other techniques they've learned for solving projectile problems.

Answers

1a 200 m, bearing 037°   **b** 14.4 ms^{-1}, bearing 304°

 c 2.83 ms^{-2}, bearing 148°   **d** 119 N, bearing 216°

2a $\mathbf{v} = \begin{vmatrix} 8 - t \\ -4 + 2t \end{vmatrix}$ (ms^{-1})   **2b i** $\begin{vmatrix} 4 \\ 4 \end{vmatrix}$ (ms^{-1})   **2b ii** North East (bearing 045°)

2c 268 N

3a $\mathbf{r} = \begin{vmatrix} 50 + 3t \\ 120 - 2t \end{vmatrix}$ (m)   **3b** $t = 60$ (seconds)   **3c** 230 metres

4a $\mathbf{s} = \begin{vmatrix} -20t + 0.1t^2 \\ 10t + 0.15t^2 \end{vmatrix}$ (m)   **4b** $\begin{vmatrix} -190 \\ 115 \end{vmatrix}$ (m)   **4c** 696 metres

5a i $\mathbf{F} = \begin{vmatrix} 0 \\ 20 \end{vmatrix}$ (N)   **ii** $\begin{vmatrix} 0 \\ 0.25 \end{vmatrix}$ (ms^{-2})   **iii** 1.5 ms^{-1} **iv** north

5b I 1 second   **5b ii** 5.25 metres

6a $\mathbf{v} = \begin{vmatrix} -3 + 0.2t \\ 2 - 0.5t \end{vmatrix}$ (ms^{-1})   **6b i** $\begin{vmatrix} -2.2 \\ 0 \end{vmatrix}$ (ms^{-1})   **ii** West (bearing 270°)

6c $\begin{vmatrix} 108 \\ -270 \end{vmatrix}$ (N)   **6d** 291 N

7a $\mathbf{v} = \begin{vmatrix} 36 \\ 12 - 9.8t \end{vmatrix}$ (ms^{-1})   **7b** $\mathbf{s} = \begin{vmatrix} 36t \\ 12t - 4.9t^2 \end{vmatrix}$ (m)   **7c** $t = 1.22$ (seconds)

7d 7.35 metres   **7e** 88.2 metres

8a i $\begin{vmatrix} 30 + 2.5t \\ 50 + 2t \end{vmatrix}$ (metres)   **ii** $\begin{vmatrix} 180 \\ 170 \end{vmatrix}$ (metres)   **8b** 426 (metres)